1. Four salespersons *A*, *B*, *C* and *D* are to be sent to visit four companies 1, 2, 3 and 4. Each salesperson will visit exactly one company, and all companies will be visited.
Previous sales figures show that each salesperson will make sales of different values, depending on the company that they visit. These values (in £10 000s) are shown in the table below.

	1	2	3	4
Ann	26	30	30	30
Brenda	30	23	26	29
Connor	30	25	27	24
Dave	30	27	25	21

(a) Use the Hungarian algorithm to obtain an allocation that **maximises** the sales. You must make your method clear and show the table after each stage.

You may not wish to use all of these lines.

	1	2	3	4
Ann				
Brenda				
Connor				
Dave				

	1	2	3	4
Ann				
Brenda				
Connor				
Dave				

	1	2	3	4
Ann				
Brenda				
Connor				
Dave				

	1	2	3	4
Ann				
Brenda				
Connor				
Dave				

(11)

(b) State the value of the maximum sales.

(2)

(c) Show that there is a second allocation that maximises the sales.

(2)
(Total 15 marks)

2. A team of four workers, Harry, Jess, Louis and Saul, are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to one task and each task must be done by just one worker.

Jess cannot be assigned to task 4.

The amount, in pounds, that each person would earn while assigned to each task is shown in the table below.

	1	2	3	4
Harry	18	24	22	17
Jess	20	25	19	–
Louis	25	24	27	22
Saul	19	26	23	14

(a) **Reducing rows first**, use the Hungarian algorithm to obtain an allocation that maximises the total amount earned by the team. You must make your method clear and show the table after each stage.

You may not need to use all of these tables.

	1	2	3	4
H				
J				
L				
S				

	1	2	3	4
H				
J				
L				

S				

	1	2	3	4
H				
J				
L				
S				

	1	2	3	4
H				
J				
L				
S				

	1	2	3	4
H				
J				
L				
S				

	1	2	3	4
H				
J				
L				
S				

	1	2	3	4
H				
J				
L				
S				

(8)

(b) State who should be assigned to each task and the total amount earned by the team.

Worker	**Task**
Harry	

Jess	
Louis	
Saul	

(2)
(Total 10 marks)

3. A company, Kleenitquick, has developed a new stain remover. To promote sales, three salespersons, Jess, Matt and Rachel, will be assigned to three of four department stores 1, 2, 3 and 4, to demonstrate the stain remover. Each salesperson can only be assigned to one department store.

The table below shows the cost, in pounds, of assigning each salesperson to each department store.

	1	**2**	**3**	**4**
Jess	15	11	14	12
Matt	13	8	17	13
Rachel	14	9	13	15

(a) Explain why a dummy row needs to be added to the table.

(1)

(b) Complete the table below.

	1	**2**	**3**	**4**
Jess	15	11	14	12
Matt	13	8	17	13
Rachel	14	9	13	15
Dummy				

(1)

(c) Reducing rows first, use the Hungarian algorithm to obtain an allocation that minimises the cost of assigning salespersons to department stores. You must make your method clear and show the table after each iteration.

You may not need to use all of these tables.

	1	2	3	4
J				
M				
R				
D				

	1	2	3	4
J				
M				
R				
D				

	1	2	3	4
J				
M				
R				
D				

	1	2	3	4
J				
M				
R				
D				

	1	2	3	4
J				
M				
R				
D				

	1	2	3	4
J				
M				
R				
D				

	1	2	3	4
J				
M				
R				
D				

(6)

(d) Find the minimum cost.

(1)
(Total 9 marks)

4. Four salespersons, Joe, Min-Seong, Olivia and Robert, are to attend four business fairs, *A*, *B*, *C* and *D*. Each salesperson must attend just one fair and each fair must be attended by just one salesperson.

The expected sales, in thousands of pounds, that each salesperson would make at each fair is shown in the table below.

	A	*B*	*C*	*D*
Joe	48	49	42	42
Min-Seong	53	49	51	50
Olivia	51	53	48	48
Robert	47	50	46	43

(a) Use the Hungarian algorithm, reducing rows first, to obtain an allocation that maximises the total expected sales from the four salespersons. You must make your method clear and show the table after each stage.

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				
R				

	A	*B*	*C*	*D*
J				
M				
O				

R				

(10)

(b) State all possible optimal allocations and the optimal total value.

(4)
(Total 14 marks)

5. To raise money for charity it is decided to hold a Teddy Bear making competition. Teams of four compete against each other to make 20 Teddy Bears as quickly as possible.

There are four stages: first *cutting*, then *stitching*, then *filling* and finally *dressing*.

Each team member can only work on one stage during the competition. As soon as a stage is completed on each Teddy Bear the work is passed immediately to the next team member.

The table shows the time, in seconds, taken to complete each stage of the work on one Teddy Bear by the members *A*, *B*, *C* and *D* of one of the teams.

	cutting	*stitching*	*filling*	*dressing*
A	66	101	85	36
B	66	98	74	38
C	63	97	71	34
D	67	102	78	35

(a) Use the Hungarian algorithm, reducing rows first, to obtain an allocation that minimises the time taken by this team to produce one Teddy Bear. You must make your method clear and show the table after each iteration.

You may not need to use all of these tables

	cutting	*stitching*	*filling*	*dressing*
A				
B				
C				
D				

	cutting	*stitching*	*filling*	*dressing*
A				
B				

C				
D				

	cutting	*stitching*	*filling*	*dressing*
A				
B				
C				
D				

	cutting	*stitching*	*filling*	*dressing*
A				
B				
C				
D				

	cutting	*stitching*	*filling*	*dressing*
A				
B				
C				
D				

	cutting	*stitching*	*filling*	*dressing*
A				
B				
C				
D				

	cutting	*stitching*	*filling*	*dressing*
A				
B				
C				
D				

(9)

(b) State the minimum time it will take this team to produce one Teddy Bear.
(1)

Using the allocation found in (a),

(c) calculate the minimum total time this team will take to complete 20 Teddy Bears. You should make your reasoning clear and state your answer in minutes and seconds.
(3)
(Total 13 marks)

6. Three workers, *P*, *Q* and *R*, are to be assigned to three tasks, 1, 2 and 3. Each worker is to be assigned to one task and each task must be assigned to one worker. The cost, in hundreds of pounds, of using each worker for each task is given in the table below. The cost is to be minimised.

Cost (in £100s)	Task 1	Task 2	Task 3
Worker *P*	8	7	3
Worker *Q*	9	5	6
Worker *R*	10	4	4

Formulate the above situation as a linear programming problem, defining the decision variables and making the objective and constraints clear.
(Total 7 marks)

7. During the school holidays four building tasks, rebuilding a wall (*W*), repairing the roof (*R*), repainting the hall (*H*) and relaying the playground (*P*), need to be carried out at a Junior School.

Four builders, *A*, *B*, *C* and *D* will be hired for these tasks. Each builder must be assigned to one task. Builder *B* is not able to rebuild the wall and therefore cannot be assigned to this task.

The cost, in thousands of pounds, of using each builder for each task is given in the table below.

Cost	*H*	*P*	*R*	*W*
A	3	5	11	9
B	3	7	8	–
C	2	5	10	7
D	8	3	7	6

(a) Use the Hungarian algorithm, reducing rows first, to obtain an allocation that minimises the total cost. State the allocation and its total cost. You must make your method clear and show the table after each stage.

(9)

(b) State, with a reason, whether this allocation is unique.

(2)
(Total 11 marks)

You may not wish to use all of these tables

	H	*P*	*R*	*W*
A				
B				
C				
D				

	H	*P*	*R*	*W*
A				
B				
C				
D				

	H	*P*	*R*	*W*
A				

B				
C				
D				

	H	*P*	*R*	*W*
A				
B				
C				
D				

	H	*P*	*R*	*W*
A				
B				
C				
D				

	H	*P*	*R*	*W*
A				
B				
C				
D				

	H	*P*	*R*	*W*
A				
B				
C				
D				

	H	*P*	*R*	*W*
A				
B				
C				
D				

8. A theme park has four sites, A, B, C and D, on which to put kiosks. Each kiosk will sell a different type of refreshment. The income from each kiosk depends upon what it sells and where it is located. The table below shows the expected daily income, in pounds, from each kiosk at each site.

	Hot dogs and beef burgers (H)	Ice cream (I)	Popcorn, candyfloss and drinks (P)	Snacks and hot drinks (S)
Site A	267	272	276	261
Site B	264	271	278	263
Site C	267	273	275	263
Site D	261	269	274	257

Reducing rows first, use the Hungarian algorithm to determine a site for each kiosk in order to maximise the total income. State the site for each kiosk and the total expected income. You must make your method clear and show the table after each stage.

(Total 13 marks)

9. Four salesperson *A*, *B*, *C* and *D* are to be sent to visit four companies 1, 2, 3 and 4. Each salesperson will visit exactly one company, and all companies will be visited.

Previous sales figures show that each salesperson will make sales of different values, depending on the company that they visit. These values (in £10 000s) are shown in the table below.

	1	2	3	4
Ann	26	30	30	30
Brenda	30	23	26	29
Connor	30	25	27	24
Dave	30	27	25	21

(a) Use the Hungarian algorithm to obtain an allocation that **maximises** the sales. You must make your method clear and show the table after each stage.
(11)

(b) State the value of the maximum sales.
(2)

(c) Show that there is a second allocation that maximises the sales.
(2)
(Total 15 marks)

10. In a quiz there are four individual rounds, Art, Literature, Music and Science. A team consists of four people, Donna, Hannah, Kerwin and Thomas. Each of four rounds must be answered by a different team member.

The table shows the number of points that each team member is likely to get on each individual round.

	Art	Literature	Music	Science
Donna	31	24	32	35
Hannah	16	10	19	22
Kerwin	19	14	20	21
Thomas	18	15	21	23

Use the Hungarian algorithm, reducing rows first, to obtain an allocation which maximises the total points likely to be scored in the four rounds. You must make your method clear and show the table after each stage.

(Total 9 marks)

You may not need to use all of these tables.

	Art	Literature	Music	Science
Donna				
Hannah				
Kerwin				
Thomas				

	Art	Literature	Music	Science
Donna				
Hannah				
Kerwin				
Thomas				

-

	Art	Literature	Music	Science
Donna				
Hannah				
Kerwin				
Thomas				

	Art	Literature	Music	Science

Donna				
Hannah				
Kerwin				
Thomas				

11. Talkalot College holds an induction meeting for new students. The meeting consists of four talks: I (Welcome), II (Options and Facilities), III (Study Tips) and IV (Planning for Success). The four department heads, Clive, Julie, Nicky and Steve, deliver one of these talks each. The talks are delivered consecutively and there are no breaks between talks. The meeting starts at 10 a.m. and ends when all four talks have been delivered. The time, in minutes, each department head takes to deliver each talk is given in the table below.

	Talk I	Talk II	Talk III	Talk IV
Clive	12	34	28	16
Julie	13	32	36	12
Nicky	15	32	32	14
Steve	11	33	36	10

(a) Use the Hungarian algorithm to find the earliest time that the meeting could end. You must make your method clear and show

(i) the state of the table after each stage in the algorithm,

(ii) the final allocation.

(10)

You may not need to use all of these grids

	Talk I	Talk II	Talk III	Talk IV
Clive				
Julie				
Nicky				
Steve				

	Talk I	Talk II	Talk III	Talk IV
Clive				
Julie				
Nicky				
Steve				

-

	Talk I	Talk II	Talk III	Talk IV
Clive				
Julie				
Nicky				
Steve				

	Talk I	Talk II	Talk III	Talk IV
Clive				
Julie				
Nicky				
Steve				

(b) Modify the table so it could be used to find the latest time that the meeting could end.
(You do not have to find this latest time.)

	Talk I	Talk II	Talk III	Talk IV
Clive				
Julie				
Nicky				
Steve				

(3)
(Total 13 marks)

1. (a) To maximise, subtract all entries from $n \geq 30$ M1

e.g $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 5 & 3 & 6 \\ 0 & 3 & 5 & 9 \end{bmatrix}$

Minimise uncovered element is 1

So $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 6 & 3 & 0 \\ 0 & 4 & 2 & 5 \\ 0 & 2 & 4 & 8 \end{bmatrix}$ M1 A2ft 1ft

M1

min. el = 2 min. el = 2

$\begin{bmatrix} 7 & 0 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 8 \end{bmatrix}$ $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$ A2ft 1ft

$A - 2 \quad B - 4 \quad C - 3 \quad D - 1$

$A - 3 \quad B - 4 \quad C - 1 \quad D - 2$ M1A1ft 2

(b) £1160 000 B2, 1, 0 2

(c) Gives other solution M1 A1 ft 2

[15]

2. (a) Since maximising, subtract all elements from some n ≥ 27

$\begin{bmatrix} 12 & 6 & 8 & 13 \\ 10 & 5 & 11 & 60 \\ 5 & 6 & 3 & 8 \\ 11 & 4 & 7 & 16 \end{bmatrix}$ 1M1 2M1

Reduce rows $\begin{bmatrix} 6 & 0 & 2 & 7 \\ 5 & 0 & 6 & 55 \\ 2 & 3 & 0 & 5 \\ 7 & 0 & 3 & 12 \end{bmatrix}$ then columns $\begin{bmatrix} 4 & 0 & 2 & 2 \\ 3 & 0 & 6 & 50 \\ 0 & 3 & 0 & 0 \\ 5 & 0 & 3 & 7 \end{bmatrix}$ 3M1 A1

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 0 & 4 & 48 \\ 0 & 5 & 0 & 0 \\ 3 & 0 & 1 & 5 \end{bmatrix}$$ 4M1 A1ft

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 47 \\ 0 & 6 & 0 & 0 \\ 2 & 0 & 0 & 4 \end{bmatrix}$$ 5M1 A1 8

Notes

1M1: Subtracting from some n ≥ 27, condone up to two errors
2M1: Dealing with (Jess, 4) entry.
3M1: Reducing rows then columns
1A1: cao (pick up (J,4) value here)
4M1: Double covered +e; one uncovered – e; and one single covered unchanged.
2 lines needed to 3 lines needed.
2A1ft: ft correct - no errors
5M1: Double covered +e; one uncovered – e; and one single covered unchanged.
3 line to 4 line solution.
3A1: correct - no errors

(b) Three optimal allocations:

Harry	3	4	4
Jess	1	1	2
Louis	4	3	1
Saul	2	2	3

M1

Total amount earned by team: £90 A1 2

Notes

1M1: A complete, correct solution.
1A1: cao

Special case (Minimises)

M0
M1

$$\begin{bmatrix} 18 & 24 & 22 & 17 \\ 20 & 25 & 19 & 60 \\ 25 & 24 & 27 & 22 \\ 19 & 26 & 23 & 14 \end{bmatrix} \xrightarrow{\textit{row reduction}} \begin{bmatrix} 1 & 7 & 5 & 0 \\ 1 & 6 & 0 & 41 \\ 3 & 2 & 5 & 0 \\ 5 & 12 & 9 & 0 \end{bmatrix}$$

M1

$$\xrightarrow{\textit{column reductions}} \begin{bmatrix} 0* & 5 & 5 & 0 \\ 0 & 4 & 0* & 41 \\ 2 & 0* & 5 & 0 \\ 4 & 10 & 9 & 0 \end{bmatrix}$$

A1

M0
M0

Solution:
Harry - 1
Jess - 3
Louis - 2
Saul - 4 M1

Total £75 A1 5

[10]

3. (a) There are more tasks than people. B1 1

(b) Adds a row of zeros B1 1

(c) $$\begin{bmatrix} 15 & 11 & 14 & 12 \\ 13 & 8 & 17 & 13 \\ 14 & 9 & 13 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0 & 3 & 1 \\ 5 & 0 & 9 & 5 \\ 5 & 0 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \rightarrow \begin{bmatrix} 3 & 0 & 2 & 0 \\ 4 & 0 & 8 & 4 \\ 4 & 0 & 3 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$ B1;M1A1

Either $$\begin{bmatrix} 3 & 3 & 2 & 0 \\ 1 & 0 & 5 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix}$$ M1 A1

Or $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 4 \\ 2 & 0 & 1 & 5 \\ 0 & 3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 5 & 3 \\ 1 & 0 & 0 & 4 \\ 0 & 4 & 0 & 2 \end{bmatrix}$$

J – 4, M – 2, R – 3, (D – 1) A1 6

(d) Minimum cost is (£)33. B1 1

[9]

4. (a) Since maximising, subtract all elements from some n ≥ 53

$$\begin{bmatrix} 5 & 4 & 11 & 11 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 6 & 3 & 7 & 10 \end{bmatrix}$$

M1A1 2

Reduce rows $\begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 3 & 0 & 4 & 7 \end{bmatrix}$ then columns $\begin{bmatrix} 1 & 0 & 5 & 4 \\ 0 & 4 & 0 & 0 \\ 2 & 0 & 3 & 2 \\ 3 & 0 & 2 & 4 \end{bmatrix}$ M1A1ft 2

Minimum element 1 M1

A1ft

$$\begin{bmatrix} 0 & 0 & 4 & 3 \\ 0 & 5 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 3 \end{bmatrix}$$

A1ft 3

M1

A1ftA1ft 3

(b)

$$\begin{bmatrix} 0 & 1 & 4 & 3 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 3 & 2 \\ 1 & 6 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$$

M1A1ft 2

M1A1 2

Joe	A	A
Min-Seong	C	D
Olivia	D	B
Robert	B	C

Value £197 000

[14]

5. (a)

$$\begin{bmatrix} 66 & 101 & 85 & 36 \\ 66 & 98 & 74 & 38 \\ 63 & 97 & 71 & 34 \\ 67 & 102 & 78 & 35 \end{bmatrix}$$

reducing rows first then columns

$$\begin{bmatrix} 30 & 65 & 49 & 0 \\ 28 & 60 & 36 & 0 \\ 29 & 63 & 37 & 0 \\ 32 & 67 & 43 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 5 & 13 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 4 & 7 & 7 & 0 \end{bmatrix}$$ M1A1

$$\begin{bmatrix} 1 & 4 & 12 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 3 & 6 & 6 & 0 \end{bmatrix}$$ M1A1ftA1ft

$$\begin{bmatrix} 0 & 3 & 11 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 1 \\ 2 & 5 & 5 & 0 \end{bmatrix}$$ M1A1ftA1ft

A – cutting
B – stitching
C – filling
D – dressing A1 9

(b) 66 + 98 + 71 + 35 =270 seconds B1 1

(c) $20 \times 98 + 66 + 71 + 35 = 2132$ seconds M1A1ft
= 35 minutes 32 seconds A1 3

[13]

6. Let $x_{ij} = 1$ if worker does task, 0 otherwise B1

where x_{ij} indicates the arc from node i to node j i.e P, Q, R j E 1, 2, 3 B1

$x_{p1} + x_{p2} + x_{p3} = 1$ $x_{p1} + x_{q1} + x_{r1} = 1$ M1

$x_{q1} + x_{q2} + x_{q3} = 1$ and $x_{p2} + x_{q2} + x_{r2} = 1$ A1

$x_{r1} + x_{r2} + x_{r3} = 1$ $x_{p3} + x_{q3} + x_{r3} = 1$ A1 3

Minimise, $C = 8x_{p1} + 7x_{p2} + 3x_{p3} + 9x_{q1} + 5x_{q2} + 6x_{q3} + 10x_{r1} + 4x_{r2} + 4x_{r3}$

where C is in hundreds of pounds B1, B1 2

B1 cao
B1 defining variable – attempt
M1 at least 3 ***equations*** *– coefficients of one*
A1 cao 3 correct
A1 cao 6 correct
B1 Minimise
B1 cao (condone a slip) (- accept cost in pounds)

[7]

7. (a) Adding $n \geq 20$ to table to give B1

	H	P	R	W
A	3	5	11	9
B	3	7	8	N
C	2	5	10	7
D	8	3	7	6

Reducing rows first $\begin{bmatrix} 0 & 2 & 8 & 6 \\ 0 & 4 & 5 & n-3 \\ 0 & 3 & 8 & 5 \\ 5 & 0 & 4 & 3 \end{bmatrix}$ then columns $\begin{bmatrix} 0 & 2 & 4 & 3 \\ 0 & 4 & 1 & n-6 \\ 0 & 3 & 4 & 2 \\ 5 & 0 & 0 & 0 \end{bmatrix}$ M1 A13

Either $\begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 0 & n-7 \\ 0 & 2 & 3 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 0 & n-7 \\ 0 & 2 & 3 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$ M1 A1ft

↓ ↓

$\begin{bmatrix} 0 & 0 & 2 & 1 \\ 1 & 3 & 0 & n-7 \\ 0 & 1 & 2 & 0 \\ 7 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 3 & 1 \\ 0 & 2 & 0 & n-8 \\ 0 & 1 & 3 & 0 \\ 7 & 0 & 1 & 0 \end{bmatrix}$ M1 A1ft 4

A	–	H		P			
B	–	R	or	R	cost £21 000		
C	–	W		H		A1	
D	–	P		W		A1	2

(b) Not unique – gives the other solution M1 A1ft 2

[11]

8. To maximise, subtract all entries from $n \geq 278$ M1

e.g.

$$\begin{bmatrix} 11 & 6 & 2 & 17 \\ 14 & 7 & 0 & 15 \\ 11 & 5 & 3 & 15 \\ 17 & 9 & 4 & 21 \end{bmatrix}$$

A1 2

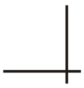
Reduce rows

$$\begin{bmatrix} 9 & 4 & 0 & 15 \\ 14 & 7 & 0 & 15 \\ 8 & 2 & 0 & 12 \\ 13 & 5 & 0 & 17 \end{bmatrix}$$

then columns

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 6 & 5 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 5 \end{bmatrix}$$

M1 A1ft A1ft 3



Min element = 1

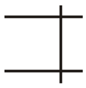
$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ 5 & 4 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 4 & 2 & 0 & 4 \end{bmatrix}$$

M1 A1ft A1 3

Min element = 1

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 5 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 4 & 1 & 0 & 3 \end{bmatrix}$$

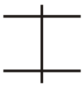
or



Min element = 2

$$\begin{bmatrix} 0 & 1 & 2 & 2 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$$

M1 A1ft A1ft 3

then  min element 1

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 4 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$$

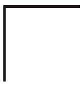
optimal

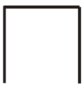
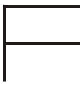
So A – H or H, B – P or S, C – S or I, D – I or P (both £1077) — M1 A1 — 2

[13]

9. (a) To maximise, subtract all entries from $n \geq 30$ — M1

e.g. $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 5 & 3 & 6 \\ 0 & 3 & 5 & 9 \end{bmatrix}$ — A2,1,0 — 3

 minimum uncovered element is 1: so $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 6 & 3 & 0 \\ 0 & 4 & 2 & 5 \\ 0 & 2 & 4 & 8 \end{bmatrix}$ — M1 A2ft1ft0 — 3

 <u>or</u>  — M1

min. el. = 2 $\begin{bmatrix} 7 & 0 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 8 \end{bmatrix}$ or min, el = 2 $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$ — A2ft1ft0 — 3

$A – 2 \quad B – 4 \quad C – 3 \quad D – 1$ — M1 A1ft — 2
$A – 3 \quad B – 4 \quad C – 1 \quad D – 2$

(b) £1160 000 — B2,1,0 — 2

(c) Gives other solution — M1 A1ft — 2

[15]

10. Subtract all terms from some $n \geq 35$, e.g.35

4	11	3	0
19	25	16	13
16	21	15	14
17	20	14	12

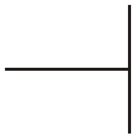
M1

A1 2

Reducing rows then columns

2	4	2	0
4	5	2	0
0	0	0	0
3	1	1	0

B1

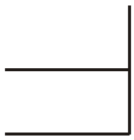


Minimum uncovered 1

1	3	1	0
3	4	1	0
0	0	0	1
2	0	0	0

M1

A1 ft 3



Minimum uncovered 1

0	2	0	0
2	3	0	0
0	0	0	2
2	0	0	1

M1

A1 ft

e.g. matching	*D - A*		*A*		*M*		*S*	A1 ft	
	H – S	*or*	*S*	*or*	*S*	*or*	*M*		
	K - M		*L*		*A*		*A*		
	T - L		*M*		*L*		*L*	A1	4

Total 88 points

[9]

11. (a) (i) Either *rows then columns* giving

	I	II	III	IV
C	0	22	16	4
J	1	20	24	0
N	1	18	18	0
S	1	23	26	0

then

	I	II	III	IV
C	0	4	0	4
J	1	2	8	0
N	1	0	2	0
S	1	5	10	0

M1, A1, A1 3

3 lines only needed ┤ (or ┬┬) least element 1 so

	I	II	III	IV
C	0	4	0	5
J	0	1	7	0
N	1	0	2	1
S	0	4	9	0

or

	I	II	III	IV
C	0	5	0	5
J	0	2	7	0
N	0	0	1	0
S	0	5	9	0

M1, A1, A1 3

Alternative

(a) (i) or *columns then rows* giving

	I	II	III	IV
C	1	2	0	6
J	2	0	8	2
N	4	0	4	4
S	0	1	8	0

(then no change)

M1, A1

3 lines only needed ┴ and *either* row 1 *or* column 3

<u>if row 1</u>: least uncovered 2

	I	II	III	IV
C	1	4	0	6
J	0	0	6	0
N	2	0	2	2
S	0	3	8	0

<u>if column 3</u>: least uncovered 1

	I	II	III	IV
C	0	2	0	5
J	1	0	8	1
N	3	0	4	3
S	0	2	9	0

Then least uncovered 1 M1 A1 M1 A1 6

	I	II	III	IV
C	0	3	0	5
J	0	0	7	0
N	2	0	3	2
S	0	3	9	0

(ii) *C* – III, *J* – I or IV, *N* – II, *S* – IV or I M1 A1
83 minutes ∴ <u>11.23 a.m.</u> M1 A1 4

(b) Subtracting all entries from some $n \geq 36$ (stated) M1
e.g. subtractions from 36

	I	II	III	IV
C	24	2	8	20
J	23	4	0	24
N	21	4	4	22
S	25	3	0	26

M1, A2,1,0 3

[13]

1. No Report available for this question.

2. This gave rise to a good spread of marks. Most candidates correctly subtracted the values from some value, n, greater than 27, although a significant minority failed to do so and minimised instead of maximising. Some failed to reduce rows and columns, or just reduced rows, or did so before subtracting the elements from n. A large number of candidates failed to deal with J4, just leaving it blank, or putting in a dash. Most then went on to apply the algorithm correctly though some arithmetical errors were seen.

3. The great majority of candidates found this an easy first question. Most candidates completed parts (a) and (b) correctly. In part (c) a number of candidates used 1, instead of 2, as the 'minimum uncovered' element in the second iteration.

4. This algorithm was generally well understood. A few candidates minimised, but most of these, realising that the solution was optimal after the row and column subtractions, restarted and maximised. Otherwise this question was extremely well done with only a few slips. Candidates should try to take care that their lines are not so thick that they obscure the numbers below.

5. This question was well done by the majority of candidates, with only a very small number reducing columns before rows or trying to maximise. A minority of candidates failed to state their final allocation. The majority of candidates failed to answer part c correctly, just multiplying their time by 20, instead of realising that work could happen concurrently.

6. Good attempts were often seen to this question, which is very pleasing since this has traditionally been an area that candidates find challenging. A few candidates treated it as a game theory problem, but most were able to set up the constraints correctly, although a small number did not use coefficients of 1. Some candidates failed to correctly define xij as being a task, *i*, allocated to a worker, *j*, and/or failed to explain the values these variables could take and why. A small number of candidates used poor notation such as p1 etc. Most candidates were able to state the objective but some failed to state that they needed to minimise the cost.

7. The vast majority of candidates did not seem know how to apply the algorithm when one worker could not be assigned to a particular job and they found a series of creative ways to deal with this situation. The two most common were to ignore this altogether, or to place a zero in the blank cell, (although the latter approach led to a solution in which a worker was allocated a task which he could not perform). The concept of assigning a 'large' value (usually at least twice the value of largest element) does not seem to be well understood. Having arrived at a solution most candidates were able state an allocation and give the cost in thousands of pounds. Part (b) was usually answered well, examiners following through from the candidate's final table, but a disappointing number merely made vague reference to the number of zeros in each row and column rather than presenting an argument.

8. This proved a good first question and a good source of marks for most candidates. A minority treated this as a minimising problem. A few made arithmetical slips. A minority did not apply the algorithm correctly with some failing to use the **minimum** number of lines at each stage and others failing to perform the appropriate addition and subjection to all relevant elements.

9. This was a rich source of marks for most candidates. Only a very few minimised. The most common errors were arithmetical slips in part (a). The units caused some problems in part (b) with candidates often writing down an answer that was a multiple of 10 out.

10. This was well done by the majority of candidates although around 20% minimised instead of maximised the score. A number of candidates did not state the final allocation or score clearly.

11. This question was usually well-answered, the biggest source of error being arithmetical slips. A few candidates did not expressly state the final matching and rather more did not convert the length of the talks into the time. Most candidates were able to complete part (b) correctly.